66TH CONFERENCE ON EXCEPTIONAL CHILDREN

Differentiating Elementary Mathematics through Cognitively Guided Instruction

Katherine Baker, UNC-Chapel Hill

11/9/16

SELF-ASSESSMENT: *A Journey of Change*

PUBLIC SCHOOLS OF NORTH CAROLINA
State Board of Education | Department of Public Instruction

November 8-10, 2016

Greetings & Introductions

- Name
- School/District
- Position

Greetings & Introductions

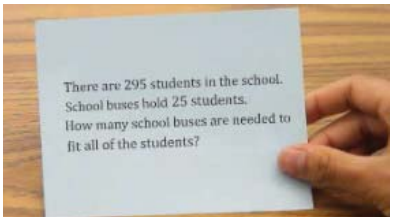
Math Talk : Would you rather be the number five or the number eight? Why?

OR

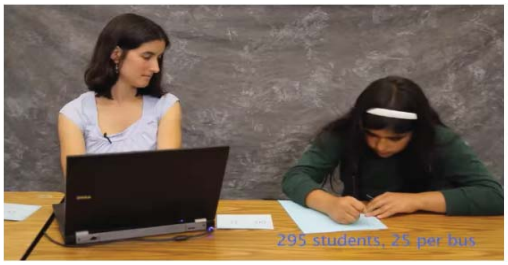
Today's Overview

- Differentiating Word Problems
  - What is Cognitively Guided Instruction (CGI)?
  - CGI addition, subtraction, multiplication, and division word problem structures
  - Overview of Student Strategies
  - CGI Classroom Teaching Structure

Instruction Through Word Problems



Instruction Through Word Problems



Avoiding Key Words

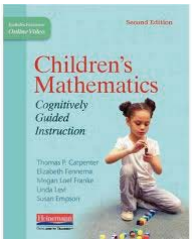
- Key words are misleading.
- Many problems have no key words.
- The key word strategy sends a terribly wrong message about doing mathematics.

A sense making strategy will *always* work.

Van de Walle & Lovin (2006)

Instead, Cognitively Guided Instruction

Meaningful instruction through word problems.



What is CGI?

- A framework for uncovering and instructing around students' ideas for solving problems.
- A variety of word problem structures teachers can use to meet all learners needs.
- An instructional style built around the belief that all students come to school with informal math knowledge and strategies and it is our job to elicit and respond.

CGI Research Has Shown…

- Children have intuitive abilities for solving math problems.
- Children develop mathematical understanding and acquire fluency by solving a variety of problems in **any way that they choose**.
- Children learn more advanced computational and problem solving strategies by watching and listening to how their classmates solve problems.

Isabella's Thinking

What grade level might use these problems?

- Tad had 15 lady bugs. He puts 3 ladybugs in each jar. How many jars did Tad put lady bugs in?
- Mr. Gomez has 20 cupcakes. He puts the cupcakes into 4 boxes so that there are the same number of cupcakes in each box. How many cupcakes did Mr. Gomez put in each box?
- 19 children are taking a mini bus to the zoo. They will have to sit 2 or 3 to a seat. The bus has 7 seats. How many children will sit 2 to a seat and how many children have to sit 3 to a seat?

Isabella's Thinking

Solve this problem.

- 19 children are taking a mini bus to the zoo. They will have to sit 2 or 3 to a seat. The bus has 7 seats. How many children will sit 2 to a seat and how many children have to sit 3 to a seat?

Discuss your strategies with an elbow partner.

Isabella's Thinking

As you watch, make note of:

- Isabella's strategies
- Teacher's strategies

http://www.heinemann.com/myOnlineResources/myVideos.aspx?sku=E05287 , 0:57

Reactions

What stood out about Isabella?	What stood out about the Teacher's moves?

- What question would you ask Isabella next?

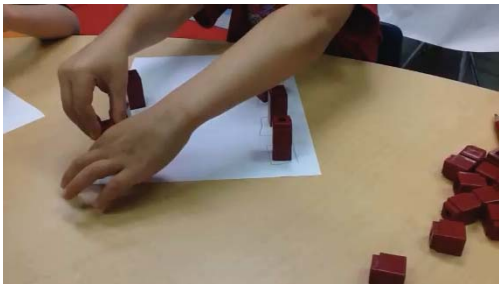
Take Note

- Reading instruction isn't taking the place of or getting in the way of the math. There's a time for reading, it may not be now.
- If a child is stuck, your first step is to repeat the problem. Give the think time instead of assuming they need help.

Let's Watch One More

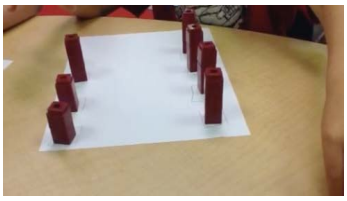
- Student in local elementary school.
- What does this student appear to be doing?

Let's Watch One More



Video Debrief

- What would you ask next?



All Children Are Capable!

- Children come into our math classrooms with a variety of ways to solve problems before we ever teach them.
- All children can make sense of math problems if we give them the time and tools to do so.

Strategic Problem Choice:
Which Problem is More Difficult?

- There are some birds in a tree. 6 flew away and now there are 8. How many birds were there to start?

OR

- 14 birds were in a tree. 6 flew away. How many birds were left?

Strategic Problem Choice

- Three parts to a typical addition/subtraction problem:
  - Start ------>Change------>Result

CGI Problem Types and Problem Sophistication

pg. 14 of book

Action	Join	*(Result Unknown)* Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have all together?	*(Change Unknown)* Connie has 5 marbles. How many more marbles does she need to have 13 marbles all together?	*(Start Unknown)* Connie had some marbles. Juan gave her 8 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
	Separate	*(Result Unknown)* Connie had 13 marbles. She gave 8 to Juan. How many marbles does Connie have left?	*(Change Unknown)* Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?	*(Start Unknown)* Connie had some marbles. She gave 8 to Juan. Now she has 5 marbles left. How many marbles did Connie have to start with?
No Action	Part-Part-Whole	*(Whole Unknown)* Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?		*(Part Unknown)* Connie has 13 marbles. 5 are red and the rest are blue. How many blue marbles does Connie have?
	Compare	*(Difference Unknown)* Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?	*(Compare Quantity Unknown)* Juan has 5 marbles. Connie has 8 more than Juan. How many marbles does Connie have?	*(Referent Unknown)* Connie has 13 marbles. She has 8 more marbles than Juan. How many marbles does Juan have?

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ACTION (explicit or implied): JOIN

JOIN RESULT UNKNOWN (JRU)	JOIN CHANGE UNKNOWN (JCU)	JOIN START UNKNOWN (JSU)
Anna has 8 fish. She wants to buy 5 more fish. How many fish would Anna have then?	Anna has 8 fish. How many more does she have to buy to have 13 fish?	

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ACTION (explicit or implied): SEPARATE

SEPARATE RESULT UNKNOWN (SRU)	SEPARATE CHANGE UNKNOWN (SCU)	SEPARATE START UNKNOWN (SSU)
11 children were playing in the sandbox. 8 children left. How many are in the sandbox now?	11 children were playing in the sandbox. Some children left. There are 3 children still playing in the sandbox. How many children left?	

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NO ACTION: COMPARE

A relationship exists among values.

Difference between is Unknown	Quantity Unknown (Looking for larger amount)	Referent Unknown (Looking for smaller amount)
CDU Jalin has 12 nickels. Lily has 7 nickels. How many more nickels does Jalin have than Lily?	CQU Lily has 7 nickels. Jalin has 5 more nickels than Lily. How many nickels does Jalin have?	CRU Jalin has 12 nickels. He has 5 more nickels than Lily. How many nickels does Lily have?

NO ACTION: PART-PART-WHOLE

A relationship exists among values.

Whole Unknown	Part Unknown
PPW-WU TJ has 8 red apples and 5 green apples. How many apples does he have?	**PPW-PU** TJ has 13 apples. 8 are red and the rest are green. How many green apples does he have?

MULTIPLICATION AND DIVISION

Multiplication (How many in all?)	Partitive Division (How many are in each group?)	Measurement Division (How many groups are needed?)
Katrina has 5 boxes of cupcakes. In each box there are 4 cupcakes. How many cupcakes does she have in all?	Karina had 20 cupcakes. She put them into 5 boxes so that there was the same number of cupcakes in each box. How many cupcakes did Karina put in each box?	Karina had 20 cupcakes. She puts them into boxes. Each box holds 4 cupcakes. How many boxes does she need?

Strategic Problem Choice

- The structure of a problem determines how challenging it is for children to solve and influences their strategies.
- Number choices also determine problem challenge level.

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The Problem Types and Problem Sophisticatio

Pg. 14 of book

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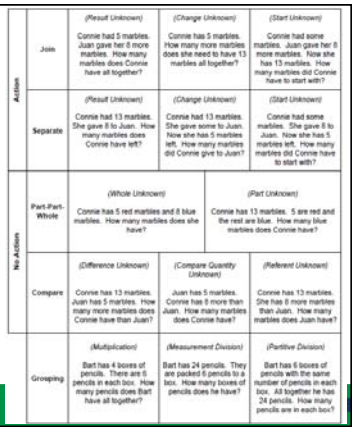
	Grouping	(Multiplication) Bart has 4 boxes of pencils. There are 6 pencils in each box. How many pencils does Bart have all together?	(Measurement Division) Bart has 24 pencils. They are packed 6 pencils to a box. How many boxes of pencils does he have?	(Partitive Division) Bart has 6 boxes of pencils with the same number of pencils in each box. All together he has 24 pencils. How many pencils are in each box?

CGI Full Chart

6TH CONFERENCE ON EXCEPTIONAL CHILDREN

- In CGI problems, you can change the NUMBERS and/or CONTEXT.
- If a new type of problem is used, consider keeping the numbers familiar.
- If new numbers are being introduced, the problem type should be familiar.



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		(Difference Unknown)	(Compare Quantity Unknown)	(Referent Unknown)
	Compare	Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?	Juan has 5 marbles. Connie has 8 more than Juan. How many marbles does Connie have?	Connie has 13 marbles. She has 8 more marbles than Juan. How many marbles does Juan have?
		(Multiplication)	(Measurement Division)	(Partitive Division)
	Grouping	Bart has 4 boxes of pencils. There are 6 pencils in each box. How many pencils does Bart have all together?	Bart has 24 pencils. They are packed 6 pencils to a box. How many boxes of pencils does he have?	Bart has 6 boxes of pencils with the same number of pencils in each box. All together he has 24 pencils. How many pencils are in each box?

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Removing the Numbers

- Connie had ____ marbles and Juan gave her ___ more marbles. How many marbles does Connie have now?

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Varying Difficulty Within Problems

- Connie had ____ marbles and Juan gave her ___ more marbles. How many marbles does Connie have now?

(5, 3) (3, 5) (8 , 4)

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Writing Our Own

- Use the CGI problem handout
- Write your own problems in your blank chart that are **relevant to your class/grade level**.

Engage students by:

- Using their names
- Using a context they all know such as a read aloud book, the classroom, neighborhood, etc.

JOIN	Result unknown	Change unknown	Start unknown
SEPARATE	Result unknown	Change unknown	Start unknown
PART-PART-WHOLE	Whole unknown	Part unknown	
COMPARE	Difference unknown	Compare quantity unknown	Referent unknown
Multiply and Divide	Multiplication	Partitive Division	Measurement Division

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Where Do I Find More Problems?

- Google search "South Dakota Counts- CGI":
- https://sdesa.k12.sd.us/esa5/docs/sdcounts/SDCountsMathProblemBooklet.pdf
- http://midcentral-coop.org/uploads/CGI%20Problems%20-%20Multiplication_Division.pdf
- And: http://www.uwosh.edu/coehs/cmagproject/cogn/word_prob.htm

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The CGI Classroom Structure

- How do we integrate these word problems into our classroom instruction?

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The CGI Classroom Structure

- *Phase 1* - Teacher presents a problem to the class
- *Phase 2* - Students spend considerable time working on solutions to the problem.
- *Phase 3* - Selected students share their strategies and thinking.

The CGI Classroom Structure

Phase 1- Introduction of Problem:

- Students are given a story problem to solve. Problem can be read several times if needed, or even acted out.
- Differentiated instruction can be utilized to meet the individual needs of each student by changing the numbers within the context.
- Ex: Various numbers choices can be given

The CGI Classroom Structure

Phase 2- Work-time:

- Students solve the problem independently without direct instruction from teacher.
- Students use any strategy of their choice.
- If finished, students are encouraged to think of more than one strategy or explain their current strategy with words.

In-the-moment Questioning Tips

During Phase 2 as you are monitoring:

- Tell me what your drawing represents.
- Where are the _____ from the story?
- Is there another way you might solve this problem? Show me.

The CGI Classroom Structure

Phase 3- Share Strategies:

- 3-5 students are selected to share their solutions and strategies with the class.
- Teacher is purposeful in who she/he chooses to share, may be a solution that moves class thinking along or challenges a misconception.
- Recommended share method is least abstract strategy to most abstract.
- Utilize your Talk Moves during this time to have discussion around strategies.

Possible Timing of Day

•Structure A:

- Warm-Up (15 minutes): A Number Talk
- Mini-Lesson (10 minutes): CGI Problem Introduction
- Work Time (20 minutes): Independent, Partners, or Small Group
- Share Time (15 minutes)

Structure B: 60 mins

- Warm-Up (20 minutes): A CGI Problem with work-time and share out.
- Your Curriculum Lesson and Work Time (30 minutes)
- Wrap-Up (10 minutes)

Note: Questioning is integrated throughout all components of your math block!

Reflect

- When might you integrate CGI problem-solving and sharing into your classroom?

Thank You!

- Katie Baker, kaphelps@live.unc.edu

References- Books

•CGI Author Books:

- Children's Mathematics: Cognitively Guided Instruction, by Carpenter, Fennema, Franke, Levi, & Empson, 1999 & 2014
- Extending Children's Mathematics: Cognitively Guided Instruction by Empson and Levi, 2010
- Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School by Carpenter, Franke, & Levi, 2003.

•AND:

- Classroom Discussions: Using Math Talk to Help Students Learn, Grades K-6 by Chapin, O'Conner,C., O'Connor, M.C., & Anderson, 2009
- Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K-5 by Parrish, 2010.

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